University of California, Berkeley
Physics 110A Spring 2003 Section 2 (*Strovink*)

Solution Set 12 (Compiled by Uday Varadarajan)

1. **Griffiths 4.10**

   (a) The surface bound charge is just $\sigma_b = \vec{P}(R) \cdot \hat{\mathbf{r}} = k\vec{r} \cdot \hat{\mathbf{r}} = kR$, while the volume bound charge is

   $$\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \vec{P} \cdot \hat{\mathbf{r}}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 k\right) = -3k. \tag{1}$$

   (b) Using the fact that we have no free charge in the region $r < R$, we know that $\vec{D}(r) = \epsilon_0 \vec{E}(r) + \vec{P} = 0$ there, so we find $\vec{E}(r < R) = -\frac{1}{\epsilon_0}\vec{P} = -\frac{k}{\epsilon_0}\vec{r}$. Now, by spherical symmetry, the field outside only depends on the total charge enclosed. This is easy to compute,

   $$Q_T = \frac{4}{3}\pi R^3 \rho_b + 4\pi R^2 \sigma_b = \frac{4}{3}\pi R^3(-3k) + 4\pi R^2(kR) = 0. \tag{2}$$

   Thus, the electric field vanishes outside, $\vec{E}(r > R) = 0$.

2. **Griffiths 4.16**

   (a) We can think of the cavity as a spherical ball of material carrying a fixed constant polarization of $-\vec{P}$ placed within the dielectric. Now, the electric field of such a spherical ball is uniform and given by equation 4.14 of Griffiths, $\vec{E}_B = -\frac{1}{3\epsilon_0}(-\vec{P})$. Thus, the electric field in the cavity is the sum of this electric field and that of the dielectric,

   $$\vec{E} = \vec{E}_0 + \vec{E}_B = \vec{E}_0 + \frac{1}{3\epsilon_0}\vec{P}. \tag{3}$$

   Since, in this region, the polarization is actually zero, we know that the displacement is just,

   $$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 \vec{E}_0 + \frac{1}{3}\vec{P} = \vec{D}_0 - \vec{P} + \frac{1}{3}\vec{P} = \vec{D}_0 - \frac{2}{3}\vec{P}. \tag{4}$$

   (b) For a needle shaped cavity parallel to $\vec{P}$, the only bound charge is associated with positive and negative charges at the ends. But if the needle is thin and long, these are small and far away from the center of the cavity, and don't really effect the electric field there. Thus, we expect that $\vec{E} = \vec{E}_0$ and $\vec{D} = \epsilon_0 E_0 = D_0 - \vec{P}$.

   (c) Finally, for a thin wafer shaped cavity, the bound surface charges on the top and bottom surfaces of the wafer are equal in magnitude to the polarization and of opposite signs, and so the wafer is just like a capacitor. Since the magnitude of the electric field of a capacitor is just $E = \frac{\sigma}{\epsilon_0}$, and points in the direction of the polarization vector, we find that the electric field in the wafer is just,

   $$\vec{E} = E_0 + \frac{\vec{P}}{\epsilon_0}. \tag{5}$$

   Thus, the electric displacement is just,

   $$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 E_0 + \vec{P} = \vec{D}_0. \tag{6}$$

3. **Griffiths 4.36**

   (a) Since we have spherical symmetry for $r > R$ in the absence of the dielectric, the candidate potential outside the conducting ball must just be a Coulomb potential with the property that $V(R) = V_0$. Thus, we must have that,

   $$V(r > R) = \frac{V_0 R}{r}. \tag{7}$$

Now, we can easily compute the electric field everywhere,

$$\vec{E}(r > R) = -\nabla V(r > R) = \frac{V_0 R}{r^2}\hat{\mathbf{r}}. \tag{8}$$

Further, we can compute the polarization in the region $r < 0$ by using the susceptibility,

$$\vec{P}(r > R, z < 0) = \chi_e \epsilon_0 \vec{E}(r > R) = \frac{\chi_e \epsilon_0 V_0 R}{r^2}\hat{\mathbf{r}}. \tag{9}$$

The bound charges densities can be computed using the polarization, noting that because the polarization is radially directed, there is no surface bound charge along the boundary of the dielectric material and vacuum for $r > R$,

$$\sigma_b(r = R, z < 0) = \vec{P}(r = R, z < 0) \cdot \hat{\mathbf{n}} = \vec{P}(r = R, z < 0) \cdot (-\hat{\mathbf{r}}) = -\frac{\chi_e \epsilon_0 V_0}{R}, \tag{10}$$

$$\rho_b(r > R, z < 0) = -\nabla \cdot \vec{P}(r > R, z < 0) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\chi_e V_0 R}{r^2}\right) = 0. \tag{11}$$

Thus, we see that not only is there no bound surface charge for $r > R$, but there is no volume bound charge there either. Finally, we can use Gauss's Law to compute the total charge density on the sphere

$$Q = \epsilon_0 \int \vec{E} \cdot d\vec{a} = 4\pi\epsilon_0 V_0 R \Rightarrow \sigma(r = R) = \frac{Q}{4\pi R^2} = \frac{\epsilon_0 V_0}{R}. \tag{12}$$

Thus, the free surface charge densities on the upper and lower halves of the sphere are given by,

$$\sigma_f(r = R, z > 0) = \sigma(r = R) = \frac{\epsilon_0 V_0}{R}, \tag{13}$$

$$\sigma_f(r = R, z < 0) = \sigma(r = R) - \sigma_b(r = R, z < 0) = \frac{\epsilon_0 V_0}{R}(1 + \chi_e). \tag{14}$$

(b) Note that the total surface charge density is σ above, which is indeed uniform, and the potential of a uniformly charged sphere is precisely,

$$V(r) = \frac{Q}{4\pi\epsilon_0 r} = \frac{\sigma 4\pi R^2}{4\pi\epsilon_0 r} = \frac{V_0 R}{r}. \tag{15}$$

(c) Since the above solution obeys all the right boundary conditions and the total charge obeys the spherical symmetry, we can apply the theorem.

(d) Figure (a) does not work the same way as the above problem since there is bound charge on the surface away from the conducting ball, as $\hat{\mathbf{n}}$ is not perpendicular to $\vec{P}$ on the dielectric boundary. However, one can solve the problem of Figure (b) in the same way since it is still the case that $\vec{P}$ is perpendicular to the dielectric boundary.

4. **Griffiths 5.56**

(a) If the angular momentum of the donut is ω, the current carried by the donut is just $I = \frac{Q\omega}{2\pi}$. If R is the radius of the donut, the magnetic moment is $m = \pi R^2 I = \frac{Q\omega R^2}{2}$. The magnitude of the angular momentum of the donut is just $L = MR^2\omega$. Thus, the gyromagnetic ratio is,

$$g = \frac{m}{L} = \frac{Q}{2M}. \tag{16}$$

(b) Since the ratio is independent of radius, and both the magnetic moment and angular momentum are found by summing over donuts of various radii, the sphere will have the same gyromagnetic ratio as any of the infinitessimal donuts it is composed of.

(c) Assuming that the electron can be modelled as a spinning solid of revolution, the magnetic dipole moment of the electron is thus,

$$m = gL = \frac{e(\hbar/2)}{2m_e} = \frac{(1.6 \times 10^{-19} C) \times (1.054 \times 10^{-34} J \cdot s)}{4 \times 9.11 \times 10^{-31} kg} = 4.61 \times 10^{-24} A \cdot m^2. \tag{17}$$

5. **Griffiths 6.12**

(a) Let's compute all the bound currents,

$$\vec{K}_b(s = R) = \vec{M} \times \hat{\mathbf{n}} = kR\hat{\mathbf{z}} \times \hat{\mathbf{s}} = kR\hat{\boldsymbol{\phi}} \tag{18}$$

$$\vec{J}_b(s < R) = \nabla \times \vec{M} = -\frac{\partial}{\partial s}(ks)\hat{\boldsymbol{\phi}} = -k\hat{\boldsymbol{\phi}}. \tag{19}$$

Now, due to the cylindrical symmetry we expect that $\vec{B} = B(s)\hat{\mathbf{z}}$ points along the $\hat{\mathbf{z}}$ direction inside the cylinder. Further, as we can think of this as infinitely many concentric solenoids, the magnetic field vanishes outside. Thus, using the usual Amperian loop for solenoids,

$$\int \vec{B} \cdot dl = B(s < R)l = \mu_0 I_{enc} = \mu_0 (\int \vec{J}_b \cdot d\vec{a} + K_b l) = \mu_0(-k(R-s)l + kRl) = \mu_0 ksl \Rightarrow \vec{B}(s < R) = \mu_0 ks\hat{\mathbf{z}}. \tag{20}$$

(b) Since we have no free currents, and as, by symmetry, $\vec{H}$ must also point in the $\hat{\mathbf{z}}$ direction, the above Amperian loop shows that $\int \vec{H} \cdot d\vec{l} = Hl = I_f = 0$, so $H = 0$. Thus, we have that $\vec{B} = \mu_0 \vec{M}$ inside, so $\vec{M} = ks\hat{\mathbf{z}}$ there while $\vec{B} = \vec{M} = 0$ outside.

6. **Griffiths 6.25**

(a) We need to compute the magnetic field due to the donut on the bottom evaluated at the position of the donut above it along the $\hat{\mathbf{z}}$ axis. As we are treating the donut magnets as dipoles with $\vec{m} = m\hat{\mathbf{z}}$, this is just Griffiths 5.86 with $\theta = 0$, $\vec{B} = \frac{\mu_0}{4\pi}\frac{2m}{z^3}\hat{\mathbf{z}}$. Now, the interaction energy of the two dipoles can be easily computed and related to the force on the upper magnet (as the magnets are back to back, $\vec{m} = -m\hat{\mathbf{z}}$),

$$U = -\vec{m} \cdot \vec{B} = \frac{\mu_0}{2\pi}\frac{m^2}{z^3} \Rightarrow \vec{F} = -\nabla U = -\frac{\partial}{\partial z}\left(\frac{\mu_0}{2\pi}\frac{m^2}{z^3}\right)\hat{\mathbf{z}} = \frac{3\mu_0}{2\pi}\frac{m^2}{z^4}\hat{\mathbf{z}}. \tag{21}$$

This upward force is balanced by gravity when,

$$F = m_d g = \frac{3\mu_0}{2\pi}\frac{m^2}{z^4} \Rightarrow z = \left(\frac{3\mu_0}{2\pi}\frac{m^2}{m_d g}\right)^{1/4}. \tag{22}$$

(b) Adding a third magnet parallel to the first and letting x be the distance between the first magnet and the second and y be the distance between the second and the third, we note that the top magnet is repelled upwards by the second while attracted downwards by the first,

$$\frac{3\mu_0}{2\pi}\frac{m^2}{y^4} - \frac{3\mu_0}{2\pi}\frac{m^2}{(x+y)^4} - m_p g = 0. \tag{23}$$

The middle magnet is instead repelled upwards by the first magnet and downwards by the third,

$$\frac{3\mu_0}{2\pi}\frac{m^2}{x^4} - \frac{3\mu_0}{2\pi}\frac{m^2}{y^4} - m_p g = 0. \tag{24}$$

Subtracting the first from the second, we get,

$$\frac{1}{x^4} - \frac{2}{y^4} + \frac{1}{(x+y)^4} = 0 \Rightarrow 2 = \frac{1}{(x/y)^4} + \frac{1}{(x/y+1)^4} \Rightarrow x/y \approx 0.85, \tag{25}$$

where we just guessed and checked to find the approximate solution.

7. **Griffiths 7.10** The induced emf is just,

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t}\left((B\hat{\mathbf{x}}) \cdot (a^2)(\cos\omega t\hat{\mathbf{x}} + \sin\omega t\hat{\mathbf{y}})\right) = -\frac{\partial}{\partial t}\left(Ba^2\cos\omega t\right) = B\omega a^2 \sin\omega t. \tag{26}$$

8. **Griffiths 7.14** We suppose the magnet is oriented such that the magnetic field above it is positive (so the bound currents which are responsible for it are moving counterclockwise). As the bar magnet falls, each ring of aluminum pipe below it experiences an increasing upward magnetic flux while each ring above it experiences an increasing downward magentic flux (note that there is no change in flux for rings along the magnet, as the magnetic field is nearly constant inside the magnet and hence no induced currents in them). The induced currents in each of these rings attempt to oppose this change in flux. Thus, the ring below will carry a clockwise current, while the ring above carries a counterclockwise current. Since opposite currents repel eachother while similar currents attract, the ring below repels the magnet upwards while the ring above attracts the magnet upwards. The net result is that the eddy currents induced by the magnet tend to delay the fall of the magnet.

9. **Griffiths 7.43**

   (a) In order to get the perpendicular component of the magnetic field to vanish at the boundary, we take a configuration in which identical poles are close to eachother and thereby repel eachother. This means that the image magnetic dipole must be oriented in the opposite direction as the original dipole, in the $-\hat{\mathbf{z}}$ direction.

   (b) The force here is just the force on the original dipole due to the image dipole, which is a distance $r = 2h$ away. This force is easily obtained by noting that $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$, where $\vec{B}$ is the magnetic field due to the image dipole (Griffiths 5.87) evaluated at the position of the original dipole, giving us,

$$F = \frac{3\mu_0}{2\pi}\frac{m^2}{(2h)^4} \Rightarrow F = Mg \Rightarrow h = \frac{1}{2}\left(\frac{3\mu_0 m^2}{2\pi Mg}\right)^{1/4}. \tag{27}$$

   (c) Using Griffiths 5.87 and adding the contributions of both dipoles at a point on the surface of the superconductor a distance r away from the origin, we find,

$$\vec{B} = -\frac{3\mu_0 mh}{2\pi}\frac{r}{(r^2+h^2)^{5/2}}\hat{\mathbf{r}}. \tag{28}$$

   The boundary conditions tell us that $\vec{B} = \mu_0(\vec{K} \times \hat{\mathbf{z}})$. Taking $\hat{\mathbf{z}}\times$ this, we find,

$$\vec{K} = \frac{1}{\mu_0}\hat{\mathbf{z}} \times \vec{B} = -\frac{3mh}{2\pi}\frac{r}{(r^2+h^2)^{5/2}}\hat{\boldsymbol{\phi}}. \tag{29}$$

10. **Griffiths 8.11**

   (a) We simply use the results for a uniformly charged, spinning shell which are given in Griffiths equation 5.68 and problem 5.36, with $\sigma = \frac{e}{4\pi R^2}$, and $m = \frac{4}{3}\pi\sigma\omega R^4$,

$$\vec{E}(r<R) = 0, \tag{30}$$
$$\vec{B}(r<R) = \frac{2}{3}\mu_0\sigma R\omega\hat{\mathbf{z}} \tag{31}$$
$$\vec{E}(r>R) = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2}\hat{\mathbf{r}}, \tag{32}$$
$$\vec{B}(r<R) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}}). \tag{33}$$

   Now, we just use the fact that the electromagnetic energy density is given by,

$$u_{em}(r>R) = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{e^2}{32\pi^2\epsilon_0 r^4} + \frac{\mu_0 e^2\omega^2}{72\pi^2 R^2}, \tag{34}$$
$$u_{em}(r<R) = \frac{\mu_0 e^2\omega^2 R^4}{18(16\pi^2)r^6}(3\cos^2\theta + 1). \tag{35}$$

Integrating over all space, we find,

$$U_{em} = \frac{1}{8\pi\epsilon_0}\frac{e^2}{R} + \frac{\mu_0 e^2\omega^2 R}{36\pi}. \tag{36}$$

(b) As both the electric and magnetic fields are non-zero only in the region $r > R$, we find a non-vanishing angular momentum density there of magnitude,

$$\ell(r > R) = \epsilon_0 \vec{r} \times (\vec{E} \times \vec{B}) = \frac{\mu_0 em}{(4\pi)^2 r^4}\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}}) \sin\theta = -\frac{\mu_0 em}{(4\pi)^2 r^4}\sin\theta\hat{\boldsymbol{\theta}}. \tag{37}$$

Integrating this over the region $r > R$ and noting that we only expect that the $\hat{\mathbf{z}}$ component will survive as $\hat{\boldsymbol{\theta}}_z = -\sin\theta$, we find,

$$\vec{L} = \int \ell d\tau = \frac{2\pi\mu_0 em}{(4\pi)^2}\hat{\mathbf{z}}\int_0^\pi \sin^3\theta d\theta \int_R^\infty \frac{1}{r^2}dr = \frac{8\pi\mu_0 em}{3R(4\pi)^2}\hat{\mathbf{z}} = \frac{\mu_0 e^2\omega R}{18\pi}\hat{\mathbf{z}}. \tag{38}$$

(c) Now, we are asked to assume,

$$L = \frac{\mu_0 e^2\omega R}{18\pi} = \frac{\hbar}{2} \Rightarrow \omega R = \frac{9\pi\hbar}{\mu_0 e^2} \approx 9.23 \times 10^{10} m/s. \tag{39}$$

Now, if we further assume that the the electrostatic interaction energy accounts for the mass and plug in the above result, we can compute R,

$$U_{em} = \frac{1}{8\pi\epsilon_0}\frac{e^2}{R}\left[1 + \frac{2}{9}\left(\frac{\omega R}{c}\right)^2\right] = mc^2 \Rightarrow R \approx 2.95 \times 10^{-11} m \tag{40}$$

Thus, $\omega = \omega R/R \approx 3.13 \times 10^{21} s^{-1}$. Clearly, since ωR, the velocity of a point on the equator of the sphere, is 300 times greater than that of light, this is a horribly unrealistic model.

11. **Griffiths 9.19**

(a) For the case of a poor conductor, we know that $\beta = \frac{\sigma}{\epsilon\omega} \ll 1$, and that the imaginary part of $\tilde{k} = k + i\kappa$ is given by Griffiths 9.126 to be,

$$\kappa = \frac{\omega s}{v} = \frac{\omega}{v}\sqrt{\frac{\sqrt{1+\beta^2} - 1}{2}} \approx \frac{\omega}{v}\sqrt{\frac{1 + \frac{\beta^2}{2} - 1}{2}} = \frac{\omega\beta}{2v} = \frac{\omega\sigma\sqrt{\epsilon\mu}}{2\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}. \tag{41}$$

The skin depth, the distance after which the amplitude decreases by factor of e, is just the distance such that the real argument of the exponential is one, which is just when,

$$\Delta z = \frac{1}{\kappa} = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}. \tag{42}$$

For pure water, $\epsilon = 80.1\epsilon_0$, $\mu \approx \mu_0$, and $\sigma \approx 1/(2.5 \times 10^5)$, so we get $d \approx 1.19 \times 10^4$ meters.

(b) The skin depth in the good conductor limit, $\beta \gg 1$, i.e. $\sigma \gg \epsilon\omega$ is,

$$\Delta z^{-1} = \kappa = \frac{\omega s}{v} = \frac{\omega}{v}\sqrt{\frac{\sqrt{1+\beta^2} - 1}{2}} \approx \frac{\omega}{v}\sqrt{\frac{\beta}{2}} = \omega\sqrt{\epsilon\mu}\sqrt{\frac{\sigma}{2\epsilon\omega}} = \sqrt{\frac{\sigma\omega\mu}{2}}. \tag{43}$$

Note that in this limit, Griffiths 9.126 shows that $k \approx \kappa$, so we have, $k \approx \sqrt{\frac{\sigma\omega\mu}{2}}$, so this means that the skin depth is really just,

$$\Delta z^{-1} = \sqrt{\frac{\sigma\omega\mu}{2}} = \frac{2\pi}{\lambda}. \tag{44}$$

For $\omega = 10^{15} s^{-1}$, $\mu \approx \mu_0$, and $\sigma \approx 10^7$ for a good conductor, we have $d \approx 13$ nanometers. Thus, light can hardly penetrate a good conductor.

(c) From Griffiths 9.134, and the fact that $\kappa \approx k$, we know $\phi \approx \tan^{-1}(1) \approx 45°$. By Griffiths 9.137, we find that the ratio of the magnitude of the electric and magnetic fields is,

$$\frac{B_0}{E_0} = \sqrt{\epsilon\mu\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}} \approx \sqrt{\frac{\sigma\mu}{\omega}}. \tag{45}$$

Evaluating this for a typical metal (i.e. for the parameters used in the previous problem), we get $\frac{B_0}{E_0} \approx 10^{-7}$ m/s.

12. **Griffiths 11.21**

(a) We can think of the oscillating charged particle as an oscillating electric dipole of magnitude $p_0 = qd$, and a frequency given by $\omega = \sqrt{k/m}$. As we are not interested in the total power radiated but in the power that hits the floor, we need to use equation 11.21 of Griffiths, the time averaged Poynting vector in spherical coordinates centered at the equilibrium position of the charge,

$$\left\langle \vec{S} \right\rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2} \hat{\mathbf{r}}. \tag{46}$$

Now, the surface we are interested in is the floor. To find the power per unit area through some point on the surface at a distance R from the point directly below the charge, we simply need to evaluate the Poynting vector at r, θ such that $\sin\theta = \frac{R}{r}$, and $R^2 + h^2 = r^2$ and dot it with the normal to the floor, which is in the $-\hat{\mathbf{z}}$ direction. Thus,

$$I_f(R) = -\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2} \hat{\mathbf{r}} \cdot \hat{\mathbf{z}} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta\cos\theta}{r^2} = \frac{\mu_0 q^2 d^2 \omega^4}{32\pi^2 c} \frac{R^2 h}{(R^2+h^2)^{5/2}}. \tag{47}$$

The radiation is most intense at the point where,

$$\frac{dI_f}{dR} = 0 \Rightarrow R = \sqrt{2/3}h. \tag{48}$$

(b) We integrate the power over the floor,

$$P = \int I_f(R) 2\pi R dR = \frac{\mu_0 (qd)^2 \omega^4}{16\pi c} \int_0^\infty \frac{R^3 h}{(R^2+h^2)^{5/2}} dR = \frac{\mu_0 (qd)^2 \omega^4}{16\pi c} \frac{2}{3} = \frac{\mu_0 (qd)^2 \omega^4}{24\pi c}, \tag{49}$$

which is exactly half the total power radiated (makes sense, the rest should hit the cieling).

13. **Griffiths 11.22**

(a) We can use methods very similar to the previous problem. As we are given the total power and the frequency of radiation, this is enough information to determine the magnetic dipole moment of the oscillating dipole Griffiths 11.40,

$$P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \Rightarrow m_0^2 = \frac{24\pi P c^3}{\mu_0 \omega^4}. \tag{50}$$

The only difference from the previous problem is that here, we are interested only in the magnitude of the radiation and not its direction, so we don't dot the Poynting vector with $\hat{\mathbf{z}}$ and instead just consider its magnitude, which is Griffiths 11.39,

$$I(R) = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \frac{\sin^2\theta}{r^2} = \frac{3P}{8\pi} \frac{R^2}{(R^2+h^2)^2}. \tag{51}$$

(b) Because this intensity drops to zero as $R \to 0$, the engineer made a completely useless measurement. It would be much more sensible to measure the intensity at its peak, which occurs at,

$$0 = \frac{dI(R)}{dR} \Rightarrow R = h. \tag{52}$$

At this location, the intensity is $I(h) = \frac{3P}{8\pi} \frac{h^2}{(2h^2)^2} = \frac{3P}{32\pi h^2}$.

(c) Plugging in the numbers,

$$I_{max} = \frac{3 \times 35 \times 10^3}{32\pi \times (200)^2} = 0.026W/m^2 = 2.6\mu W/cm^2. \tag{53}$$

14. **Griffiths 11.25**

We can think of the particle and its image charge as a dipole with dipole moment given by $p(t) = 2qz(t)$, where $z(t)$ is the distance between the dipole and the conducting surface. Now, the force on the charge due to its image is just,

$$F = m\ddot{z} = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{4z^2} = -\frac{\mu_0 c^2 q^2}{16\pi z^2} \Rightarrow \ddot{p} = 2q\ddot{z} = -\frac{\mu_0 c^2 q^3}{8\pi z^2}. \tag{54}$$

The total power radiated can now be found by using equation 11.60 of Griffiths,

$$P = \frac{\mu_0 \ddot{p}^2}{6\pi c} = \left(\frac{\mu_0 c q^2}{4\pi}\right)^3 \frac{1}{6m^2 z^4}. \tag{55}$$

15. Consider a pair of coaxial cables, the first filled with a dielectric ϵ_1, the second filled with a dielectric ϵ_2, both with inner radius a and outer radius b.

(a) What is the capacitance per unit length of each cable? Suppose we put free charge Q on a length l of the inner conductor. Then,

$$\int \vec{D} \cdot d\vec{a} = \epsilon E 2\pi s l = Q \Rightarrow E = \frac{Q}{2\pi\epsilon s l} \Rightarrow V = -\int_b^a \vec{E} \cdot d\vec{l} = \frac{Q}{2\pi\epsilon l} \ln\left(\frac{b}{a}\right). \tag{56}$$

Thus, the capacitance per unit length is just,

$$C/l = \frac{Q}{Vl} = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}. \tag{57}$$

(b) What is the inductance per unit length of each cable? It turns out that the easiest way to do this is to use a trick outlined in example 7.3 of Griffiths. The point of that section was to note that the inductance could also be computed using the fact that the energy stored in the magnetic fields of an inductor is $\frac{1}{2}LI^2$. However, as we know the magnetic field in the coaxial cable, $\vec{B} = \frac{\mu I}{2\pi s}\hat{\phi}$, we can compute this energy directly by integrating the energy density $\frac{1}{2\mu}B^2$ over all space to find $W/l = \frac{\mu I^2}{4\pi}\ln\left(\frac{b}{a}\right)$. Thus, comparing with $\frac{1}{2}LI^2$ we have,

$$L/l = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right). \tag{58}$$

(c) Suppose the two cables are connected at a junction. If a TEM wave is sent through this junction, what fraction of the power of the wave is transmitted? This is a bit of a trick question. Essentially, as TEM modes have the same dispersion relation as flat space modes, and as the relation between the magnetic and electric fields are also the same, applying the boundary conditions of Griffiths 9.74 on passage between the two cables yields the same result as the flat space plane-wave. We find, just as in that case, that for normal incidence,

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}. \tag{59}$$

where $n_i = \sqrt{\frac{\epsilon_i}{\epsilon_0}}$.

16. Suppose we have two infinite grounded conducting plates parallel to the xz plane, one at $y = 0$ and the other at $y = a$. Close off the left end (at $x = 0$) by an infinite metal strip which is insulated from the other conducting plates. Consider a confined TEM mode propagating in the z direction in this "waveslot" and describe, up to a multiplicative constant, its electric field profile at $z = t = 0$.

The point here is just that for a TEM mode, the electric field profile at fixed z and t is just a solution to the 2D Laplace equation in the directions transverse to the direction of propagation (x and y) with the boundary conditions specified as above. In particular, these boundary conditions are precisely those solved in Example 3.3 of Griffiths. I will refer you to that section, as the answer is just $\vec{E}(x,y) = -\nabla V(x,y)$ with $V(x,y)$ given by Griffiths 3.36.